EPICYCLIC GEARING AND THE ANTIKYTHERA MECHANISM
PART II

by M.T. Wright

PART I of this article appeared in March 2003.[26] I withdrew the concluding part because more refined data became available. I apologize to readers for the long-delayed completion of the consequent revision.

The Antikythera Mechanism, dateable to the first century BC, is by far the oldest geared mechanism in the world. I began by showing that all earlier attempts to understand it were vitiated by the acceptance of mistaken arrangements described by Professor Derek Price, on whose writing most readers' understanding of the Mechanism is (directly or indirectly) based. I described how, working from new observations of the original artefact made by the late Professor A.G. Bromley and myself, I was able to correct one of Price's errors and so to develop a new reconstruction of the dial on one face (Price's *front dial*) as a planetarium. I introduced epicyclic gearing to model the motions of the Sun, Moon and planets, taking as a precedent the epicyclic system, found in the original fragments, which Price had identified as a differential gear.

Adopting that one correction alone would have led to a difficulty with the supposed differential gear, which forms part of the train leading from the wheelwork under the front dial to the lower back dial on the opposite face of the instrument, but a second correction resolved this. Where Price postulated the existence of two inputs to the epicyclic cluster, in reality there is only one; so the assembly is not a differential gear, but an epicyclic gear with a stationary central wheel, followed by a fixed-axis train. The scheme is familiar to students of complicated dial-work as one adopted where a ratio is desired that cannot conveniently be achieved by a fixed-axis train alone, and I suggest that this is the reason for its introduction here. It is a little startling to find the application in so early an instrument, but arguably it is no more so than the differential gear that it 'supplants'.

We know that one revolution of the input to this train represented one tropical month, but the poor state of preservation of the instrument makes it hard to be sure what output period was intended. The numbers of teeth in several of the train wheels are uncertain and we cannot even be sure of the number of arbors on the epicyclic platform, while the remaining part of the lower back dial offers few direct clues as to the function displayed on it. In part I, I showed that we must abandon the detail of this part of Price's gearing scheme because it does not accord with what can be seen of the original train. I also suggested that its principal output (a period of one synodic month) was implausible: it must have been clear to the designer that this output period, *exactly* consistent with the ratio between year and tropical month realized in the wheelwork serving the front dial, could be obtained using a very simple scheme.[27] His use of epicyclic gearing clearly implies that he had some other purpose.

The detail of the instrument matters far less than the simple fact that such a highly-developed artefact was designed and executed at all, at so early a date. Nevertheless, this unique survival of a tradition of fine mechanism, predating the

26. *Antiquarian Horology*, 18/3 (March 2003), 270-79. The footnotes, figures and tables of this second part are numbered to run consecutively with those of the first.

27. The period-relation is 19 years = 235 synodic months. The designer's appreciation of this relationship is implicit in his train to the upper back dial, which I have now elucidated: with its velocity ratio of 3.8:1, and input period of 1 year, its output period is 47 synodic months; that is, the pointer makes five revolutions in 235 months, or 19 years. See: M.T. Wright, 'Counting Months and Years', *Bulletin of the Scientific Instrument Society*, no.86 (September 2005), forthcoming.

introduction of clockwork by perhaps 1300 years, merits the horologist's attention; and even the limited evidence available will bear considerable development. Here I explore the function of the epicyclic gear and the fixed-axis train that leads from it to the lower back dial.

GEAR WHEEL ANALYSIS

We rely on radiography for a clear view of the wheel work within the corroded fragments of the Antikythera Mechanism, and so Bromley and I prepared a large number of high-quality X-ray images. The instrument is, however, constructed on a small scale.[28] Few of the wheels are complete, and of some very little remains. Direct examination of the radiographs over a light-box, using a magnifier, is both tedious and exacting.

Subsequently it became feasible to have radiographs scanned and digitized, with a view to further manipulation. Bromley took our plates to Sydney to have this done, but the equipment available proved inadequate and he abandoned the project when he fell seriously ill. I recovered the plates only later, and had them scanned in the summer of 2003.[29] Thereafter any one of 689 images, stored as digitized files, could simply be called up. The image on the screen can be magnified and its brightness and contrast can be modified at will, making the inspection of detail much easier and more certain. The Cartesian coordinates of any point in the image can be captured at the click of a mouse-button and transferred into a spreadsheet program for analysis.

These computer tools were first used in order to reappraise each of the toothed wheels, superseding such methods as overlaying the radiograph with transparent templates comprising circles of different sizes and with different numbers of radial divisions, and inspecting for coincidence.

I have described my procedure in outline,[30] and a more detailed account is in preparation.[31] Meanwhile I have published a summary of my results.[32] From that paper I abstract the tooth-counts for the wheels that are germane to the present discussion (Table 2) and the diagram of my gearing scheme (Fig. 9), which supersede and extend the information given in Table 1 and Fig. 8 of part I of this paper. Some of the detail of Fig. 9 pre-empts conclusions that I shall reach here or adumbrates points that will be discussed in future papers.

It will be seen that in some cases the new analysis leads, paradoxically, to greater uncertainty as to the numbers of teeth in the wheels. The main reason for this lies in the irregularity of division of the wheels, which had previously escaped attention. This interesting fact will form the basis of further discussion, but for now I note only that the variations in pitch show insufficient system to be of much help in reducing the uncertainty in estimating the tooth-counts of some of the mutilated wheels.

Column 1 of Table 2 gives the designation of each wheel in Price's scheme and in my extension of it. In columns 2 and 3 I give respectively the tooth-counts of the radiographer Karakalos, as reported by Price, and those adopted by Price. Columns 4 and 5 comprise my results. Where I give a single value in column 4 and none in column 5, either I can find all the teeth or I can find enough to be quite sure of the full count. Otherwise I offer in column 4 a *preferred* count, based mainly on the assumption that the wheel was divided as uniformly as the observed data points will allow, and in column 5 I give a wider *range* of possibilities, consequent mainly on allowing the mean pitch of the restored

28. The largest wheel, roughly 130 mm. in diameter, seen prominently in Fig. 1 (part I), had between about 216 and 231 teeth, corresponding to about 0.56 to 0.60 module. All the other wheels, ranging down to a pinion of 15, have smaller teeth.

29. I am indebted to Professor (now Sir) Mike Brady, Head of Department, for allowing this to be done in the Medical Vision Laboratory, part of the Information Engineering Department of the University of Oxford, and to my son Dr G.J.T. Wright for scanning my plates and for setting up tools for the analysis of digitized images on my computer.

30. M.T. Wright, 'The Scholar, the Mechanic and the Antikythera Mechanism', *Bulletin of the Scientific Instrument Society*, no.80 (March 2004), 4-11.

31. M.T. Wright & G.J.T. Wright, *Computer-Aided Analysis of Radiographic Images, applied to the Antikythera Mechanism*, in preparation.

32. M.T. Wright, 'The Antikythera Mechanism: a New Gearing Scheme', *Bulletin of the Scientific Instrument Society*, no.85 (June 2005), 2-7.

TABLE 2

	Karakalos	Price	Wright preferred	Wright range
E3	192	192	191	188 – 192
E5	50 to 52	48	53	51 – 55
E8			51	50 – 52
F1	54	48	54	53 – 55
F2	30	30	30	
G1	20	20	20	
G2	54/55	60	55	54 – 55
H1	60 to 62	60	60	57 – 64
H2	16	15	15	
I	60	60	60	59 – 60
K2	48 or 51	48	49	48 – 50
K3	[48 or 51]		49	48 – 50

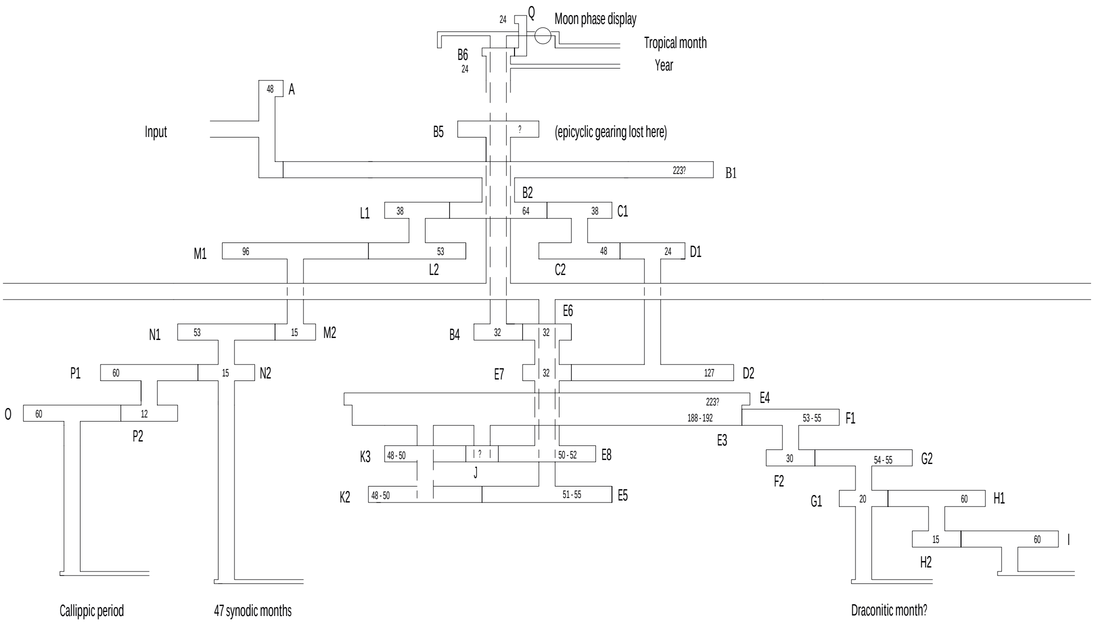

Fig. 9. Gearing scheme for the Antikythera Mechanism.

teeth to vary by up to about 5% of the mean of the surviving ones. It seemed impossible to set these limits wholly objectively: judgment was exercised, for example, as to how trustworthy individual data points might be, and as to just how far the division of the wheel might have departed from uniformity while remaining workable in its particular place in the scheme. The reader should therefore be aware that in at least some instances the actual number of teeth might have lain even outside the stated limits.[33]

33. Without information about the depth of engagement, the loads transmitted and other factors which cannot be determined, it is impossible to say with certainty just what variation in pitch would be tolerable while still allowing the instrument to be worked. For that matter, we have no guarantee that it did ever work in a satisfactory way.

TABLE 3

	wheels								Output periods at axis G in days	
	E3	F1	F2	G2	E5	E8	K2	K3	with idler	without idler
Preferred tooth-counts	191	54	30	55	53	51	49	49	28.878531	-0.555356
Tooth-counts chosen to	192	53	30	54	51	52	50	48	26.357442	
yield extreme values of	188	55	30	55	51	52	50	48		0.856696
output at axis G	188	55	30	55	55	50	48	50	31.445056	-2.137043

RECONSTRUCTION OF THE GEAR TRAIN

The epicyclic cluster is broken across, and so we have only about half of the platform and of each of the wheels on it. Therefore we face particular difficulty in making accurate tooth-counts for these wheels, and especially for the smaller ones. The fragmentary peripheries of some of them seem not very precisely circular, and the tooth-count of such a 'half-wheel' is very sensitive to the choice of the centre. Besides, on each axis we find two wheels, lying one over the other, rather close to one another in size and tooth-count, which makes observation difficult; and, as will be explained below, the centres of **K2** and **K3** do not coincide.[34]

For these reasons, the tooth-counts for wheels **E5**, **E8**, **K2** and **K3** cannot be given with any great degree of confidence, and so the velocity ratio of the epicyclic cluster and the rotational period of the platform are not very closely determined. This uncertainty is reflected in that of the output period at the dial; but there is another, more radical, source of difficulty.

Price's reconstruction of the epicyclic cluster as a differential gear required the connection of one of the central wheels to the corresponding wheel on the epicyclic axis **K** through an idle wheel, to reverse the sense of rotation. Accordingly, he suggested the presence of the two well-separated small wheels **E2ii** and **K1**, with the idle wheel **J** planted on the epicyclic platform between them (fig. 5, part I). However, Price's small wheel **K1** simply does not exist, and his **E2ii** is a misinterpretation of the trace of wheels **E6** and **E7** in my corrected scheme (Fig. 9), in which the wheels corresponding to Price's **K1** and **E2ii** (moved to the opposite face of the platform, to accord with reality) are designated **K3** and **E8**. Wheels **E5**, **E8**, **K2** and **K3** are all very similar in size, and they are pitched so that the wheels in both the upper and the lower pairs, **E5** & **K2** and **E8** & **K3** respectively, approach one another very closely.

Thus the state of the original is such that observation of the engagement between these fragmentary wheels is very difficult. It appears that the spatial relationship between them has been disturbed, and that they have become crushed into one another. It is now extremely hard to be sure whether only one pair was originally engaged or whether both pairs were. In other words, on the basis of the physical evidence it is uncertain whether there was an idle wheel.[35] Since the epicyclic cluster was not a differential gear, it might in principle have functioned either with or without the idle wheel; and so we must consider both possibilities.

For each case, the ranges given in Table 2 for the tooth-counts of wheels in the train from axis **E** to axis **G** give rise to 4050 permutations to be tried. A spreadsheet program was set up to make the calculation for all permutations and to select and sort those results that fell within any chosen error bound for the output period. A first selection of results is shown in Table 3.

34. Price reports that Karakalos was quite uncertain about the count of his wheel **E5** where, with hindsight, we see that he was counting **E5** and **E8** together, and that he saw Price's **K2** as a 'double wheel' (as, indeed, it was: **K2** and my **K3**, lying together). It should be remembered that the schematic diagrams, Figs 5, 6, 8 & 9, do not render the wheels to scale.

35. By the same token, it is unclear whether, if we are to restore an idle wheel, it should be placed between **E8** and **K3** or between **E5** and **K3**. The choice between the two is, however, of no consequence for the overall function of the assembly.

TABLE 4

E3	F1	F2	G2	E5	E8	K2	K3	Output in days
190	53	30	54	55	50	48	50	-2.000607
191	54	30	55	55	51	50	48	-0.499821
192	53	30	54	54	52	50	48	0.041771

In the last column are periods obtained on the assumption that the epicyclic gear had no idle wheel, while in the penultimate column are those corresponding to the presence of an idle wheel, all expressed in days.[36] In the first row are those corresponding to the *preferred* values for the tooth-counts, column 4 of Table 2. In the following three rows are those that yield the extreme values of the output period got by exploring the results of the full *ranges* for the wheels, as listed in column 5 of Table 2.

The velocity ratio for the epicyclic gear, or number of turns of the platform (the output) for every turn of wheel **E8** (the input), may be written as follows:

$$V = 1/\{1 \pm (E5 \times K3)/(E8 \times K2)\},$$

where $E5$ denotes the number of teeth in wheel **E5**, and so on. The operator '±' is to be taken as '+' if we suppose that there was an idle wheel, and '–' if we suppose there was not. In what follows it should be remembered that since wheels **E5**, **E8**, **K2** and **K3** were all rather similar in size and tooth-count, the quantity $(E5 \times K3)/(E8 \times K2)$ was close to unity.

Firstly we consider the behaviour of the arrangement *without* an idle gear. In this case we have a high velocity ratio, with the platform turning very fast relative to the input. (Note that $(E5 \times K3)/(E8 \times K2)$ could not have actually equalled unity because this would have corresponded to an infinitely large velocity ratio, a calculated output period of zero; in mechanical terms, the input would have been locked while the platform would have been free to rotate.) The periods in the last column of Table 3 are correspondingly short, ranging from – 2.137043 through zero to + 0.856696 days. The negative sign against an output period indicates rotation in the sense opposite to the input.

Several striking results are listed in Table 4: output periods close to two days and to half a day; and – most remarkable of all – the period 0.041771 day which is close to 1/24 day, and very close indeed to 1/24 sidereal day. Here we have the surprising possibility that the lower back dial might have been read like the dial of a regulator clock, the main and subsidiary pointers having periods representing one hour and twelve hours respectively.

With any of these short output periods it is, however, important to look beyond the kinematics of the arrangement. The step-up ratio through the epicyclic gear, from input **E8** to output **E3**, would be large; in the last case, the ratio from input wheel **E8** to arbor **F** would be over 1177:1! With the crude gear teeth and other mechanical details of this instrument, even less extreme versions of this arrangement could work only if driven from near the fast-moving end. The two most plausible options are that the instrument might have been worked by turning the pointer on arbor **G** – either by hand or through some external mechanical arrangement – or that a lost driving train might have ended in a pinion that engaged the (unexplained) gear teeth on the edge of the epicyclic platform (**E4**).[37] Especially with the reconstruction in which the pointer at **G** might make one turn

36. These figures are based on the assumption that one turn of the date pointer on the front dial is taken as a year of 365¼ days, which is supported by the fact that the calendar ring is laid out according to the Egyptian calendar of 365 days, with provision for moving it by one day every four years in relation to the astronomical events, according to the convention of that calendar. Output periods are rounded to 6 decimal places throughout.

37. Cf. the arrangement adopted by Bromley, in a conjectural reconstruction devised before either he or I had inspected the original: a two-stage driving train worked by an input turning approximately once a day leads to a pinion engaging **E4**. See: A.G. Bromley, 'Notes on the Antikythera Mechanism', *Centaurus*, 29 (1986), 5-27; A.G. Bromley, 'Observations of the Antikythera Mechanism', *Antiquarian Horology*, 18/6 (Summer 1990), 641-52. Our subsequent investigation caused Bromley to abandon this reconstruction, privately at least.

per hour, the temptation is to imagine that the Antikythera Mechanism might have been driven by a water-clock; but, so far as we know, even the largest and most carefully made sundials included no graduations to subdivide the hour, so that it is hard to understand what interest a 'minute hand' could have held. Moreover, in my opinion the instrument is built on too small a scale for it to have been the designer's intention that it should have been driven by a clock. Rather, I see the instrument as an essay in miniaturization that was intended to be portable. Besides, since I have shown that Price's *Sun Position* wheel (Fig. 5) cannot have existed, if the instrument were driven by turning a fast-moving arbor in this train we would then have no function for the contrate wheel **A**, which can be understood only as the means of driving the instrument (Fig. 9). This point will be considered together with other evidence suggesting that the instrument may have been altered, but on another occasion.

Even if we suppose that at one time the instrument had a fast-moving train to the lower back dial which was driven more or less directly, the design of the back dial itself does not seem to support the suggestion that the day or any other short period might have been displayed on it. As I will describe, the arrangement of the dial suggests an interest in periods of four turns of the main pointer taken together, and this main pointer moved over a scale divided into about 55 parts – certainly fewer than 60 – equipped for the placing of moveable markers. Meanwhile the subsidiary dial, with few markings and with its pointer turning at one-twelfth of the rate, can best be understood as a means of keeping count of the cycles of four turns of the main pointer. These details make no sense in relation to any of the short-period solutions. It is also to the point to ask what possible purpose an indication such as the day, the half-day or the hour could have served on an instrument that was not driven as a clock. I conclude that we face an impasse over the reconstruction of the epicyclic gear with no idle wheel.

We now consider the behaviour of the arrangement *with* an idle wheel. In this case, the velocity ratio for the epicyclic gear must be close to 1:2. The range of possibility given in part 1 for the velocity ratio of the fixed-axis train leading from the epicyclic assembly to the pointer at the centre of the lower back dial on axis **G** (wheels **E3**, **F1**, **F2** and **G2**) must be broadened to conform to the revised estimates of the tooth-counts of the damaged wheels shown in Table 2, but it remains close to 2:1. It follows that, if the epicyclic gear included an idle wheel, the overall ratio from input (**E8**) to the pointer at the centre of the lower back dial on axis **G**, had to be close to unity. We now have a complete and *workable* train from the driving wheel **A**, through the wheelwork under the front dial of the instrument, to the lower back dial, even if we remain unsure of the actual numbers of teeth in some of its wheels.

We know that **E8** made one turn in a tropical month. Table 3 shows that, by letting the tooth-counts run through the *range* of tooth-counts listed in column 5 of Table 2, the extreme values for the output period at the lower back dial (axis **G**) are 26.357422 and 31.445056 days. Between these limits, the only possibility appears to be that the intended output was also some type of month. We have already ruled out the synodic month. The tropical month and the sidereal month are numerically so close that it seems highly unlikely that one would attempt to set up gearing to relate one to the other in such an instrument as this; besides, it is actually improbable that the two were seen as clearly distinct at the time when the Antikythera Mechanism was designed.[38] The remaining possibilities are the draconitic month (27.212219 days) and the anomalistic month (27.554571 days).[39]

The anomalistic month may be ruled out. There is good reason to incorporate the anomaly into a display modelling the Moon's place in the Zodiac (as I have done in my reconstruction of the front dial), but the Moon's place with respect to its apogee, which is what a display of the anomalistic month would show, is of little direct interest. On the other hand, a display of the draconitic month, which is an expression of the place of Moon with respect to its nodes, may be used in the prediction of eclipses. When the Moon is at syzygy (as could be seen on the front

38. The distinction between the two depends on an understanding of the *precession of the Equinox*.

39. These figures are taken from O. Pedersen, *A Survey of the Almagest*, (Odense University Press, 1974), Appendix B, as 'values implicit in Ptolemaic astronomy, but not actually quoted in the Almagest'.

TABLE 5

E3	F1	F2	G2	E5	E8	K2	K3	Output in days
188	53	30	54	51	52	50	49	27.190194
191	53	30	55	51	51	50	48	27.242698
192	53	30	54	52	55	48	51	27.212796

dial, either to a poor approximation in Price's reconstruction or more precisely in mine) and is sufficiently near to a node (at the end or mid-point of a draconitic month), the appropriate eclipse can be predicted.

I conclude therefore, by elimination, that one turn of the principal pointer on the lower back dial represented one draconitic month, the only attainable function that could serve a useful purpose. Eclipse-prediction, based on such simple criteria, is at best uncertain, and there is no need, in short-term prediction, to adopt a particularly close approximation for the draconitic month. There is a choice of numbers within the ranges given in Table 2, column 5, that yield an output period close enough to 27.21… days to be satisfactory. Two examples are given in the first and second rows of Table 5. The first is the more precise, but the reader will see that the tooth-counts leading to the second conform a little more closely to the set of *preferred* numbers. This second result is such that the pointer would be out of place by just over 2½° in 6½ draconitic months, a period within which an eclipse-possibility commonly repeats. As mentioned above, the actual tooth-counts could have lain outside the limits given in column 5 of Table 2, and by widening the scope just a little one may obtain a much more accurate period for the draconitic month. The example given in the third row of Table 5 was derived in a different way that is explained below.

Just conceivably, the epicyclic platform might have carried a compound train instead of a simple idle wheel. It is now difficult to be certain of the original relationship between the levels of the remaining wheels on axes **E** and **K**, and so one might imagine a wheel-pair on the stud at **J** instead of a single idle wheel. One might go further, and imagine a train of several pairs, perhaps up to three, even within the distinctly limited space. This could widen somewhat the possible range of periods given in the penultimate and the last columns of Table 3 (depending on whether there were an odd or an even number of axes). However, in either case the geometry of the arrangement suggests strongly that the velocity ratio of such additional wheelwork could not have been far from unity. Therefore its introduction could probably not lead to any further possible function for the train, but only to some refinement to its velocity ratio. This would be gained at the cost of considerable extra friction that might well bring the assembly to the verge of impracticability. It is proper to draw attention to the idea, but I am not encouraged to pursue it further. On the basis of the available evidence, I favour a reconstruction in which the epicyclic platform carried a single idle wheel, interposed between either wheels **E5** and **K2** or **E8** and **K3**.

THE DESIGN OF THE EPICYCLIC GEAR

There seems to be little evidence as to how an epicyclic gear might have been designed at the time when the Antikythera Mechanism was made. Here I offer a sketch illustrating a naïve approach, which depends on finding of a fixed-axis train offering a rough approximation to the desired velocity ratio and then modifying it by adding an epicyclic cluster. Very possibly the method actually used was more sophisticated.

The output period at the centre of the lower back dial was to be the draconitic month, and the input period to the train was the tropical month. A period-relation between the two had to be established. For the sake of illustration, consider the period-relation now commonly called the Saros, which was certainly well known:

$$242\ T_d = 223\ T_s$$

The following further relation is implicit in the other gearing preserved in the instrument:

$$235\ T_s = 19 \text{ years} = 254\ T_t$$

(Note: T_d = draconitic month; T_s = synodic month; T_t = tropical month.)

Combining these two, we have:

$$(235 \times 242)\ T_d = (223 \times 254)\ T_t$$

and the required velocity ratio, with numerator and denominator reduced to prime factors, can be expressed as:

$$(5 \times 11 \times 11 \times 47) \div (127 \times 223)$$

Beginning with the approximation built into the mechanism under the front dial, whereby the tropical month is 365¼ × (19/254) = 27.321850 days, this leads to a good value for the draconitic month, of 27.212313 days.

The original fragments of the mechanism include wheels having numbers of teeth from 15 up to about 223, both factorable and prime. It is therefore clear that this output, if wanted, might have been achieved using one of several fixed-axis trains embodying the given velocity ratio exactly. It follows that the designer probably did not start from this pair of period-relations but worked from data that led to a less tractable ratio; but whatever his starting point the numerical value of the ratio would have been quite similar.

In any case, he would then have searched for manageable approximations to the velocity ratio. Perhaps he found the following one:

$$479/477$$

which is good to 1 part in 6250. Now 477 = (3 × 3 × 53), while 479 is prime, but 480 can be factorized in many convenient ways. As one example, one might write:

$$480/477 = (96 \times 30)/(53 \times 54)$$

Suppose that the designer chose to embody this less good but more easily manipulated ratio in a fixed-axis train, meaning to improve the approximation by adding an epicyclic cluster, and that his starting-point in designing the latter was to imagine having equal wheels on the platform. As noted above, this 'basic' epicyclic cluster has a velocity ratio of 1:2, so the designer might have begun by doubling the velocity ratio of the fixed-axis train. He would in any case have needed a large number for the teeth on the edge of the epicyclic platform. So he might have obtained:

$$(192 \times 30)/(53 \times 54)$$

a set of numbers which the reader may recognize by reference to wheels **E3**, **F2**, **F1** and **G2** (in that order) in Table 2 and Fig. 9. Thus, if the designer began with the approximation 479/477, and changed the prime number 479 to 480 because it is the product of small prime factors, he could easily have been led to this set of numbers for the fixed-axis train which is compatible with the result of my gear analysis. The point remains valid even if the actual tooth-counts were slightly different: they were probably chosen so as to bear a direct relationship to some convenient approximation to the desired ratio.

Now the approximation 480/477 is too large, by about 1 part in 442. It would have remained for the designer to attempt to find a set of numbers for wheels **E5**, **E8**, **K2** and **K3** such that the velocity ratio of the epicyclic gear, $1/\{1 \pm (E5 \times K3)/(E8 \times K2)\}$, was lower than ½ in a corresponding proportion; that is, $(E5 \times K3)/(E8 \times K2)$ = 443/441, approximately. In keeping within the range of tooth-pitch used in the mechanism as a whole, the designer would have thought it convenient to have roughly 50 teeth in each of the four wheels in question. Beginning with $50^2 = 2500$, and through a reasonably short routine that could have been devised intuitively and worked without any difficulty, he could find sets of numbers to fulfil the requirement. One comes quite quickly to the following good approximation:

$$(52 \times 51)/(55 \times 48) = 2652/2640\ [= 221/220]$$

This offers a set of tooth-counts for **E5**, **E8**, **K2** and **K3**. The numbers within each bracket may be taken in either order, but whichever way they are taken two of them lie a little outside the ranges given in Table 2. The permutation that conforms most nearly to those ranges is shown in the third row of Table 5. The accuracy of the corresponding output period, 27.212796 days, is such that the pointer would remain within 1° of its correct position for over 131 draconitic

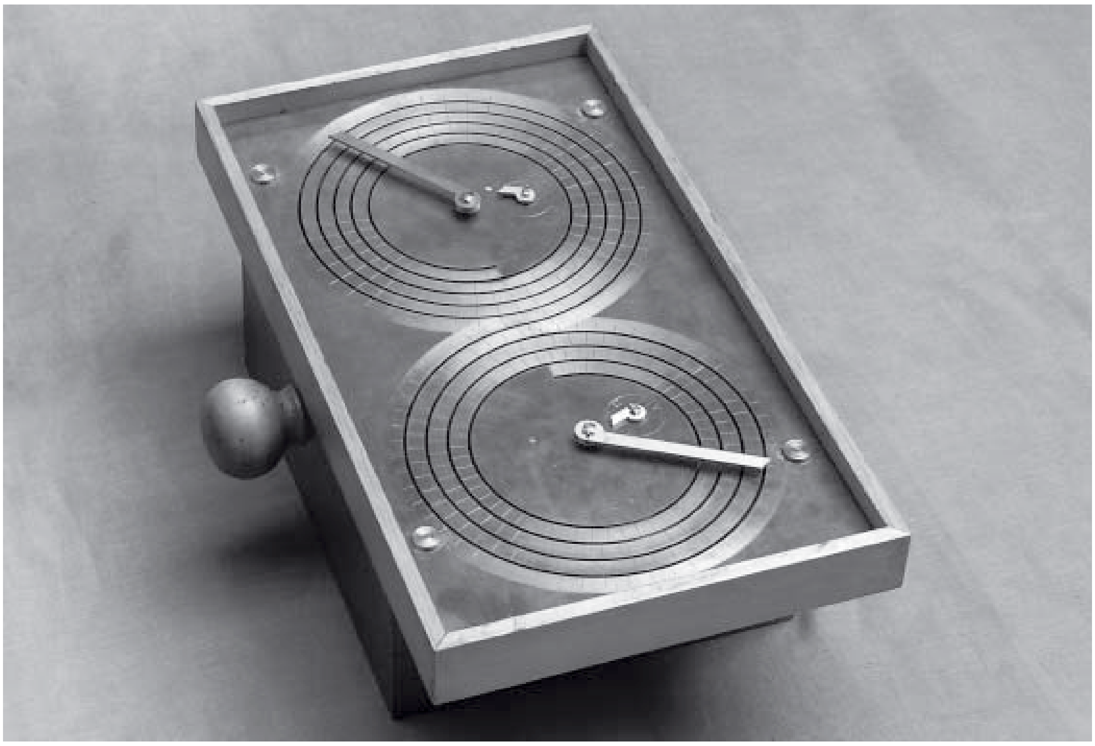

Fig. 10. The Antikythera Mechanism, reconstruction by the author of the back dial. The epicyclic train leads to the lower dial system.

months. An output still closer to the true value for the draconitic month can be obtained if the ranges for the tooth counts of some or all of the wheels are broadened just a little further. I did not fix those limits in any very rigorous way, but there is little point in speculating too earnestly in this direction because in reality we know neither with what period-relation or other data the designer began nor how he worked out his design.

It is not easy to explain how the maker missed, or why he passed over, the possibility of a train developed from the Saros period-relation; or, equally, other approximations from which less precise but handier fixed-axis trains could have been formed, which still yield better results than does any fixed-axis-plus-epicyclic train that I have devised. Thus, while we may celebrate the designer's inventiveness in adopting an epicyclic train, it seems possible that his powers of arithmetical analysis were limited. In any case the adoption of epicyclic gearing suggests a misplaced preoccupation with accuracy; even if the Moon's distance from a node (the end or mid-point of a draconitic month) at syzygy is found to great precision, the prediction of an observable eclipse on the basis of this criterion alone remains uncertain.

THE DIAL DISPLAY

The computer tools that were devised for use with gear wheels have also been applied to analysis of the geometry of the remaining fragments of the dial. For brevity I again state only my conclusion. The slots around both the upper and the lower back dials seem to have consisted not of systems of concentric circles but of continuous spirals, of five and four turns respectively. The evidence appears consistent with 'spirals' of the simplest possible design, comprising semicircular arcs struck from two centres on the vertical mid-line, one above the other. The bridge-pieces riveted to the back of the dial to span the slots were clearly designed to allow the passage of rivet-heads around the slots, showing that the function of the slots was to contain moveable marker beads or segments. Possibly the outer turns of the two spiral systems were joined, making a single double-spiral slot. These details are illustrated in Figs 10 & 11 by photographs of my reconstruction of the dial plate.

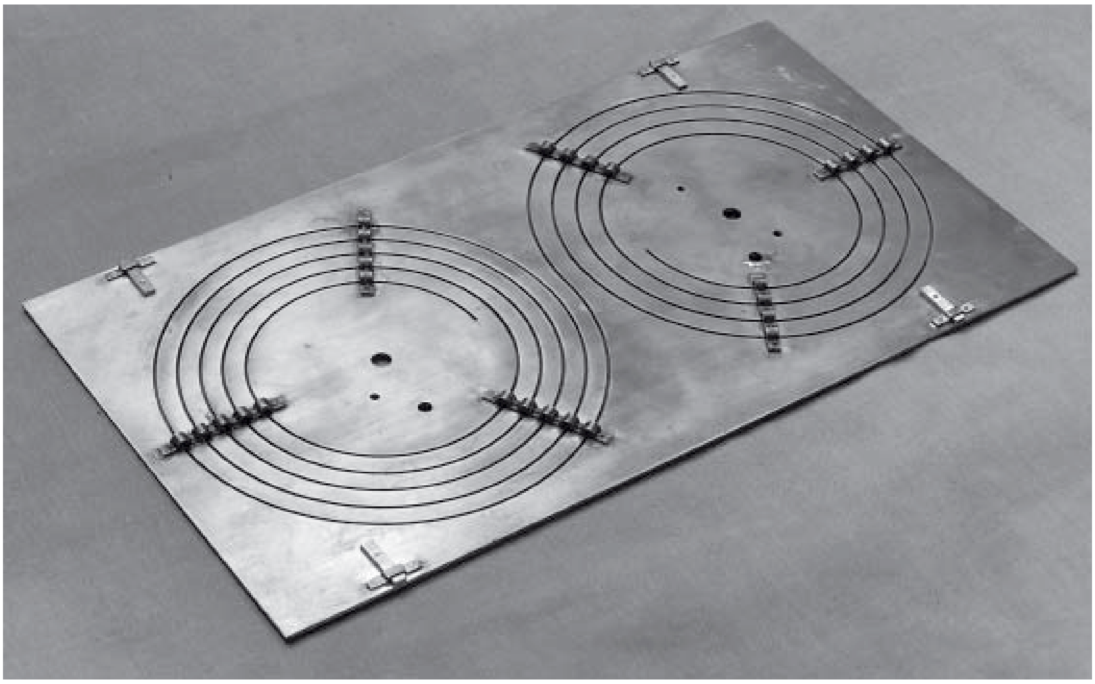

Fig. 11. The Antikythera Mechanism, reconstruction by the author. Inside of the back dial to show the bridge-pieces that connect the turns of the spiral systems.

The graduations along the slots of the original fragments can be traced over only rather short arcs, and so the number of divisions in the full circle is uncertain. The evidence is consistent with there being 54½ divisions per turn in the lower dial system. If one turn represented one draconitic month, each division would correspond to a half-day, to a fair approximation. The four-turn spiral scale would contain 218 divisions, 109 whole days. This may explain the choice of a scale representing four draconitic months, a period of no obvious intrinsic astronomical significance.

The figures for the tooth-counts of wheels **G1**, **H1**, **H2** and **I** (Table 2) show that the pointer of the subsidiary dial turned once in about 12 turns of the main pointer. In fact, its indication makes sense only as a way of keeping count of longer periods than that shown on the main dial; in other words, the periods of the two pointers must have been commensurate, with the ratio exactly 12:1, as Price has it and as I show in Fig. 9. Incidentally, this deduction offers an object-lesson in the danger of trying to determine the numbers of teeth in mutilated wheels by geometrical analysis alone. We are forced to conclude that wheel **I** had 60 teeth, while the analysis suggests rather confidently that it should have had 59; we are obliged to restore 14 teeth to an arc that is only a little too large to accept 13 of the mean size of those that remain.

The subsidiary dial appears to have been divided into three. Price reported reading one character within its circle, but I read two more, the three disposed at roughly 120° to one another. The pointer on this dial would therefore show the transition from one cycle of four draconitic months to the next. The longer period of 12 draconitic months, which is represented by the use of main and subsidiary dials together, is one within which an eclipse-possibility is bound to recur.

The procedure for using this display in eclipse prediction might have been somewhat as follows. The user could set the instrument to the times of syzygy at which eclipses had been observed, using the front dial. (The Moon and Sun pointers would lie over one another for New Moon, and stand opposite one another for Full Moon.) In each case he could mark the place of the pointer on the lower back dial using a moveable marker. Using the front dial again, he might then set the instrument to the date of a further syzygy. He could then compare the position of the pointer on the lower back dial

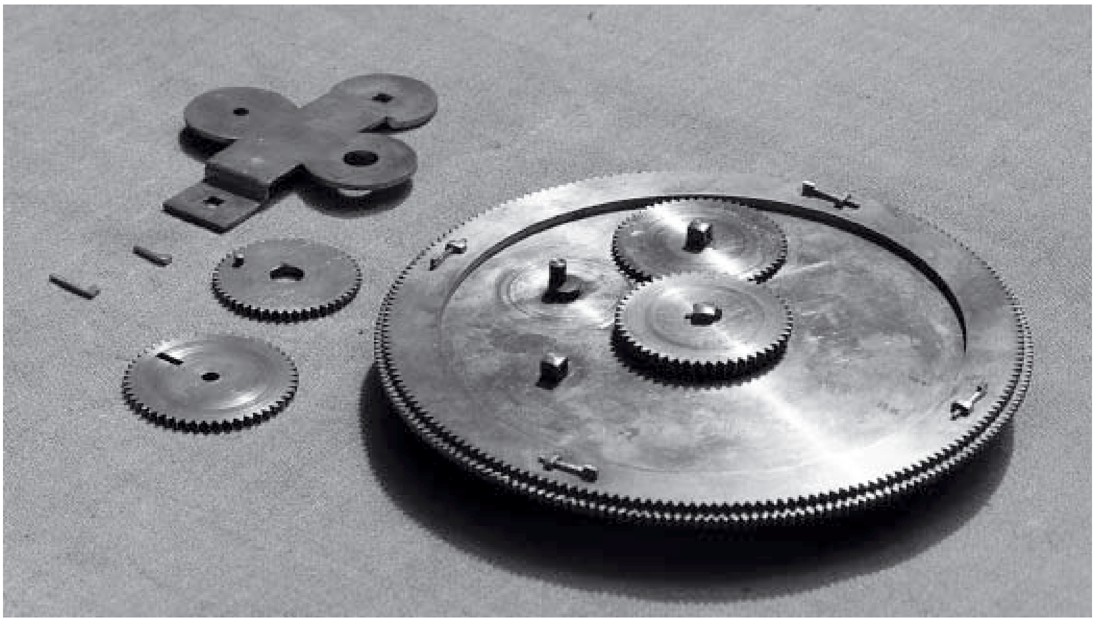

Fig. 12. The Antikythera Mechanism, reconstruction by the author. The epicyclic gear, partly disassembled to show the pin-and-slot coupling between the wheels on axis K.

with that of the previously-set markers. In effect, he would be examining how closely syzygy fell at the end or mid-point of the draconitic month; that is, how near the Moon was to a node. At New Moon, an eclipse of the Sun is certain if the angle is within 13½°, and possible if it is within 18½°; at Full Moon, an eclipse of the Moon is certain if the angle is within 9°, and possible if it is within 12½°.[40] The scale graduations, at intervals of about 6½°, would serve for judging these criteria. Assuming that the user knew the time of an observed eclipse, the half-day graduations might have guided him in judging the time of the predicted eclipse, which would of course have influenced the likelihood of observing it.

UNRESOLVED PROBLEMS AND THE QUESTION OF MODIFICATION

Price reported a curious feature found within the fragmentary epicyclic gear, but his observation was inadequate and his interpretation of it mistaken. There is a radial slot in wheel **K2** which, at the outer end, corresponds to the loss of a tooth. Towards the centre the slot has clean-cut, parallel sides and a square end. Price suggested that it had been cut to house a slip, shaped at the outer end to replace a damaged tooth. Close examination shows, however, that the slot was originally closed at the outer end; its present appearance is due to the loss of a small piece that once closed the end of the slot and included the missing tooth. Radiographs show the circular trace of a pin planted in wheel **K3** directly below the slot. The pin engaged in the slot compelled wheels **K2** and **K3** to turn together although they were not rigidly fixed to one another; and the reason for avoiding a rigid fixing is found in the central detail that is again seen only by radiography: the two wheels are mounted on a stepped stud, so that they turn about different centres.

The arrangement is best illustrated by photographs of my model. In Fig. 12 the components on axis **K** are shown separated to reveal the stepped stud, pin and slot. In Fig. 13 they are shown assembled. The wheels, riding on their separate steps, are kept in place by the guard piece that lies over them.

A comparable arrangement of mobiles turning about separate centres on a stepped

40. These figures are taken from Bromley's paper (note 37). Bromley discusses the possibility of a display similar to the one that I discuss here, but occupying the upper back dial of a rather wild variant of Price's reconstruction which is not supported by the artefactual evidence.

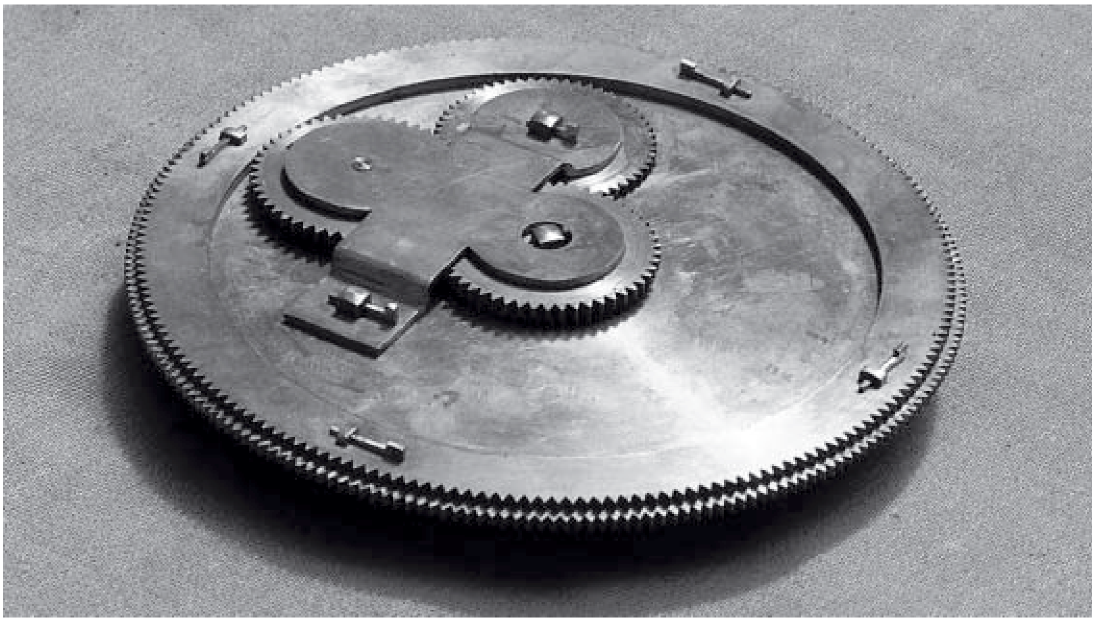

Fig. 13. The Antikythera Mechanism, reconstruction by the author: the epicyclic gear, assembled.

stud is sometimes found in dial-work modelling Ptolemaic planetary theory,[41] but the device can serve no such purpose here. This instrument was lost at sea some two centuries before the date at which Ptolemy introduced the *equant* to planetary theory, and it is this feature that necessitates the fitting of mobiles on several closely-spaced centres when a working model of the theory is to be made. In any case, such a use could not explain the presence of the ensemble on an epicyclic axis. Moreover, the arrangement of the epicyclic cluster here, behind the back dial and away from any of its centres, rules out any possibility that it was intended to model a geometrical arrangement.

The effect of the ensemble is to introduce a roughly sinusoidal wave into the velocity ratio of the train, which could have been intended to model an *anomaly* of astronomical theory, and the only possibility in this train, concerned with months, is lunar theory. However, the wave's period is determined by the rate of rotation of the wheels relative to the epicyclic turntable, which is roughly twice the period of rotation of the main pointer on the lower back dial, two months, and its amplitude, found by dividing the offset between the axes of wheels **K2** and **K3** by the radius at which the pin is set in wheel **K2**, is roughly 12% of the mean value. Neither period nor amplitude can be matched to the anomaly of Hipparchus's lunar theory (which we know as the 'first anomaly' of Ptolemy's theory), the only known candidate for the era of the instrument.

The presence of this device does not, of course, change the mean velocity ratio of the train, and so it does not alter the foregoing argument concerning its output period, but its purpose here remains an unresolved problem in my reconstruction of the Antikythera Mechanism. It does however provide a happy precedent for the use of slotted levers embracing eccentrically-mounted pins, which I have adopted in my reconstruction of the front dial. This is, indeed, the earliest evidence both for the use of the crank pin and for the combination of crank pin and slotted follower. As such, it represents a highly significant moment in the history of technology.[42]

41. The device was used, for example, by de' Dondi.

42. Price (note 2) made a tentative identification of another component as a 'folding crank handle' for working the instrument, an identification that seems to have become more certain in the minds of later writers. I have previously shown that this component cannot have been any such thing: M.T. Wright, A.G. Bromley & H. Magou, 'Simple X-ray Tomography and the Antikythera Mechanism', PACT 45 (1995) (Proceedings of the Conference 'Archaeometry in South-Eastern Europe' held in Delphi 19th. – 21st. April 1991, 531-43.

The problems of the epicyclic cluster are compounded by the unexplained presence of gear teeth on the edge of the epicyclic platform (**E4**). If the 'probable' count of 223 teeth is correct, then this wheel could have formed part of a fixed-axis train based on the Saros period-relation. Earlier, I pointed out that the factorized ratio

$$(5 \times 11 \times 11 \times 47) \div (127 \times 223),$$

derived from the Saros, might have formed the basis of such a train in place of that including the epicyclic gear. Note that if the true count for **E3** were 188, it would contain the factor 47. The wheel-pair **E3/E4** could then be understood as a reused relic from such a fixed-axis train that was, for some unknown reason, rebuilt to include the epicyclic gear.

There are further details of the fragments which all point strongly to the possibility that the instrument underwent extensive rebuilding. In that case other ensembles within it may now be found in contexts for which they were not originally designed. These interesting points, which seem to me to place the Antikythera Mechanism more firmly within an *extended tradition* of instrument-making, will be discussed on another occasion.

CONCLUSION

It is important to stress that the confidence with which any reconstruction of the Antikythera Mechanism can be offered must be tempered by the limitations of the evidence supporting it. In particular, one should resist the temptation to suggest that certain wheels 'must' have had certain numbers of teeth because the gearing scheme then provides a plausible function for the instrument, if the conjecture is insufficiently supported by other evidence. What I have done here, for the gear train that includes the epicyclic gear, is more rigorous. I have worked from new observations of the original fragments which are more accurate than those of Price; they are based both on direct examination and on radiographs that reveal much more of the internal arrangement than those available to him. For the train leading to the lower back dial, I have confined myself to tooth-counts within the ranges of what my analysis of those radiographs shows to be possible. Setting aside for the present the possibility that the picture has become confused through alteration of the instrument, I find only one solution that seems to be readily workable, is consistent with what can be deduced about the design of the dial display that it supported, and bears a plausible relationship to the other indications of the instrument and to the known astronomical preoccupations of its time. It is that the principal output period at the lower back dial was an approximation to the draconitic month. From this I conclude that the dial display was arranged to be used in eclipse-prediction.

Figure 9 shows details that I have not yet discussed, either here or in the paper from which it is abstracted, and I have mentioned in passing other details on which I have more to say. It must be clear that there is a great deal still to be written about the Antikythera Mechanism, although much of it may not be of prime interest to the readership of this journal. What will interest the horologist is the fact that this instrument is evidence for mechanical design of a high order drawing on a wide repertoire of mechanical ensembles (including epicyclic gearing, if not the differential gear), and that it was executed with assurance and skill. It is therefore hard to imagine that it does not represent a tradition of fine mechanism, applied to astronomical modelling and perhaps to other ends, which was already established by the first century BC. The discovery of the London Byzantine Sundial-Calendar tends to support the idea that the tradition remained alive in Hellenistic culture and was transmitted to Arabic culture.[43] There seems to be no evidence that such dial-work was ever linked directly to the contemporary water-clock, but supposing that this tradition of astronomical model-making was transmitted thence to Latin Europe alongside the documented transmission of astronomical knowledge itself, we may imagine that it was already known in Europe as the Western tradition of clock-making arose. Hence we need not be surprised by the early flowering of clocks with elaborate astronomical dials.

43. J.V. Field & M.T. Wright, *Early Gearing*, (London: The Science Museum, 1985).